## A LATE SUNDIAL AT APHRODISIAS

(Plate II)
During the excavations at Aphrodisias in Caria several ancient sundials have come to light. Most are only fragmentary, but one, standing as a pedestal in a paved area of the city (the post-scaenam piazza) is substantially in one piece and of considerable interest.

The dial (plate II) is in the form of a cylindrical pillar of marble, the upper surface of which is horizontal and stands approximately one metre above ground level. The top is slightly larger than the main body of the column because of a lip and was originally an oval of diameters $88 \cdot 5$ cm (east-west) and 78 cm (north-south) though the south side is broken away for perhaps one fifth of its original width. The base of the dial, $40-50 \mathrm{~cm}$ below pavement level, also has a similar lip, which betrays its earlier use, which was as an altar. ${ }^{1}$ The religious symbols have been erased-unless the crescent within a circle on its north side is one (but see below). Into the horizontal upper surface are incised quite bold lines of width approximately 3 mm , three running east to west, two being hyperbolae, ${ }^{2}$ and the one between a straight line, and eleven lines crossing from one hyperbola to the other in a general north-south direction.

This was a horizontal dial calculated to show the usual temporary or seasonal hours and of a pattern known from several Greco-Roman examples. Fifteen other such dials from different parts of the Greco-Roman world are cited by Gibbs. ${ }^{3}$ The gnomon cast a shadow across the pattern of lines and gave the time by the position of its tip in relation to the north-south grid of hour lines and the date according to the east-west lines, the northern hyperbola being the shadow-path on the day of the winter solstice, the southern that at the summer solstice, and the straight line that on the days of the equinoxes. A hole 3.2 cm across immediately south of the point where the equinoctial line crosses the meridian (the line running due north-south) marks where the gnomon, probably a bronze or iron rod set in lead, if other Greco-Roman dials are any guide, ${ }^{4}$ was fixed on the meridian. The gnomon itself may have been slightly inclined, ${ }^{5}$ but the shadow of its tip was what indicated the time, so that the lean, if any, and position of the point of fixing are not important to the working of the dial. Along the winter solstitial curve (to be read from the south side) is inscribed XIMEPINH(sc. $\tau \rho o \pi \dot{\eta}$ ), on the summer solstitial $\Theta E P[I] N H$, and divided between the two extremities of the equinoctial line is $I C H M E / P I N H$. The style of this lettering suggests that the dial was cut between the fourth and sixth centuries A.D., probably the fifth. Apart from this the only other inscription on the pillar is a symbol of a circle with a crescent inside it on the northern side near the top. The motif of the crescent (perhaps the symbol of the diallist?) is found on some other ancient dials. ${ }^{6}$

The Aphrodisias dial has been rather badly damaged, especially around the gnomon area. The examples in Gibbs' catalogue give a clear picture of the dial type, which is rightly identified by her with that described as the pelecinum, or 'double-axe' (from the pattern of its grid of hour

[^0]moving. The plane of the horizon cuts through this at an angle equal to the complement of the latitude of the place of observation.
${ }^{3}$ S. L. Gibbs, Greek and Roman Sundials (New Haven/London 1976) (hereafter Gibbs) 78, 323-38, cat. nos. 440IG-4015 (reviewed by the present writer, $C R$ xxviii [1978] 336-9).
${ }^{4} C f$. Gibbs 79.
${ }^{5}$ Ibid. Cf. also Cetius Faventinus De Diversis Fabr. Arch. 29 (ed. V. Rose, p. 310, l. 9)—paululum inclinis ponitur. See below.
${ }^{6} C f$. Gibbs 89.
lines), by Vitruvius and Cetius Faventinus. ${ }^{7}$ The present specimen is curious in that computation of its constructional latitude suggests that this was different from that of Aphrodisias by as much as seven degrees, the width of the eastern Mediterranean. The grid of hour and date lines which makes up the pelecinum is of a slightly different shape according to the latitude of the place for which the construction was made. The overall size of the dial-type is dependent upon the height of the gnomon (the spike which casts the shadow), but the proportions vary with latitude, so that the further north from the equator that the constructional latitude is, the further away from the gnomon are the markings. Since the sun is always lower in the sky at any given moment in a higher latitude than it is in a lower one, the shadows cast on a horizontal surface are in consequence longer. Unfortunately the damage to the stone prevents those measurements outlined by Gibbs ${ }^{8}$ from being taken in order to test the constructional latitude and derive the size of the gnomon. The marks which remain around the meridian area of the stone are not very clear, but are crucial to the calculation of these two quantities. In the discussion of the dial which follows these measurements have been used as far as possible according to how much can be read from the stone. Clearly no great accuracy is in any case possible, when the incised lines are so substantial as 3 mm .


Fig. I.

In order to discover the intended constructional latitude of the dial, we can proceed in the following way. Consider FIG. I. A is the tip of the gnomon, vertically above the meridian BDE at B. For the sake of clarity we are here seeing the dial from the north-west, while in PLATE IIIc it is viewed from the south. The metal rod of which the tip A once cast the shadow is represented by AY. AB is thus the effective height of the tip of the gnomon (A) above the dial face.FDO is the path of the shadow of A at the equinoxes (the equinoctial) at right angles to BDE at D. F, G, H, I, J, K, L, M, N, O are the positions of the shadow of A, as marked by the hour lines on the stone at the seasonal hours from first $(\mathrm{F})$ to fifth $(\mathrm{J})$ and seventh $(\mathrm{K})$ to eleventh ( O ) (the sixth hour being noon, D ) on the day of the equinox. C and E are the positions of the shadow of A at midday at the summer and winter solstices respectively, and also lie on the meridian. AD and AE are the sun's noon rays through A at the equinox and winter solstice respectively. AD and AM and all lines joining A to points on FDO lie in the plane of the celestial equator, an imaginary circle in the sky, the projection of the earth's equator around which the sun appears to move throughout the day of the equinox in any latitude. Since the sun is above the horizon for half of the day at the equinox, an equinoctial or equatorial hour, by definition, will equal a seasonal or

7 Vitruv. De Arch. ix 8. i: Patrocles pelecinum dicitur invenisse-see Gibbs 61; Cetius Faventinus (n. 5) ch. 29,

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tinus', CQ xxix (1979) 203-I2.
    8 Gibbs 4I f., 323.
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temporary hour, which is $1 / 12$ of the period from sunrise to sunset on any day in the year. Hence, for example, if $M$ marks the position of the ninth hour, or three hours since noon, the sun will have moved through $9 / 12$ of its half-circle around the celestial equator from sunrise to sunset, or $135^{\circ}$; since noon falls at $90^{\circ}$ from sunrise, this point will be $45^{\circ}$ from noon. Similarly the angular distance of any point marked on FDO is $15^{\circ}$ from its neighbour, measured at A. This angle between any point on FDO and D is commonly called the Hour Angle. In the triangles formed by the points on FDO, A and D, the angle at A is therefore always known and the distance from D to the point on FDO can, in most cases, be measured on the stone. ${ }^{9}$ Table i shows the figures for these positions.

Two points should be borne in mind in connection with this Table: (i) that the lines on the stone have a definite width, and consequently accuracy of measurement cannot be expected any more than accuracy of cutting; (ii) that in Table I the differences in the measurements of the points on FDO are irregular. The sun's apparent movement during any day at any place on the earth is symmetrical about the meridian in comparison with the horizon, and similarly the lines on a horizontal dial should be symmetrical about the noon (sixth seasonal hour) line.

|  | Temporary <br> Hour | Angle <br> at A | Distance of <br> point from D | Values for <br> AD |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | cm | cm |
| F | I | $75^{\circ}$ | 28.50 | 7.64 |
| O | II |  | 29.20 | 7.82 |
| G | 2 | $60^{\circ}$ | 12.70 | 7.33 |
| N | IO |  | 13.00 | 7.51 |
| H | 3 | $45^{\circ}$ | 7.40 | 7.40 |
| M | 9 |  | illegible | - |
| I | 4 | $30^{\circ}$ | 4.40 | 7.62 |
| L | 8 |  | 4.50 | 7.79 |
| J | 5 | $15^{\circ}$ | illegible | - |
| K | 7 |  | illegible | - |

Table I

If we take the lengths of $A D$ implied by each hour point on the equinoctial line FDO it is possible within certain limits to achieve a reasonable idea of the correct position of $A$. In each case the length of AD is found by multiplying the distance of the hour point on FDO from D by the cotangent of the angle at A .

For example, point F , seasonal hour I ,

$$
\begin{aligned}
& \frac{\mathrm{AD}}{\mathrm{DF}}=\frac{\mathrm{AD}}{28.5}=\cot 75^{\circ} \\
& \text { therefore } \mathrm{AD}=28.5 \times \cot 75^{\circ}
\end{aligned}
$$

The values derived in this way for AD are given in the final column of table i.
One further measurement can be taken from the dial, but the broken condition of stone around the centre makes it difficult to read accurately. This is DE, taken as 5.30 cm . To solve FIG. I one more angle is needed, DÂE, which is the difference between the angular altitude of the midday sun at the equinox (AD) and the winter solstice (AE) and corresponds to the Inclination of the Ecliptic ( $\epsilon$ ) or the sun's maximum (southerly) declination ( $-\delta$ ). In antiquity this was

[^1]slightly greater than it is today and was generally held to be a round figure of $24^{\circ}$ (i.e. one fifteenth of a circle). ${ }^{10}$

Table i has shown certain differences in the derived lengths of AD, the greatest being nearly half a centimetre. These discrepancies for AD must be taken into account in our calculations based upon it; an average value of $A D$ is not sufficient. It is better to take a set of round values ranging from just below the lowest postulated length of $\mathrm{AD}, 7.33 \mathrm{~cm}$, to almost its highest, 7.82 cm . We have therefore used six figures for $A D$ at intervals of Imm . To find the latitude for which the dial was calibrated (or intended) and the effective height of the gnomon, AB, consider FIG. 2, which consists of the central elements of FIG. I, basically the meridian line (BDE), the effective gnomon, AB , and two rays of sunlight passing through A , the tip of the gnomon; AD is the line taken by such a ray at noon (seasonal hour 6) at the equinoxes and AE a ray at the same time of day at the winter solstice. The actual metal rod which formed the gnomon is shown as AY, but, since we cannot measure where Y was in relation to B, AY and YB could have been any convenient length. As we have said above, it is the height of $A B$ which determines the layout of the lines on the dial, when taken in combination with the latitude.

To solve fig. 2 we must first imagine a perpendicular from D to AE at X . From this follow the formulae below.
I. To find the latitude $\left(\epsilon=24^{\circ}\right)$ :

In the triangle $A X D$,
$\mathrm{XD}=\mathrm{AD} \times \sin 24^{\circ}$
and in the triangle EXD, $\mathrm{XD}=\mathrm{DE} \times \sin \mathrm{XED}$ therefore
$\mathrm{AD} \times \sin 24^{\circ}=\mathrm{DE} \times \sin \mathrm{XE} D$
$\sin \mathrm{XE} D=\frac{\mathrm{AD} \times \sin 24^{\circ}}{\mathrm{DE}}$
By the angle sum of triangles and of angles on a straight line
XÊD $=A$ ÊD
$\mathrm{ADE}=180^{\circ}-\left(24^{\circ}+\mathrm{AED}\right)$
BDA $=180^{\circ}-$ ADE
BD̂A $=180^{\circ}-\left(180^{\circ}-\left(24^{\circ}+\right.\right.$ AED $\left.)\right)$
BD̂A $=24^{\circ}+\mathrm{AÊD}$
BÂD $=90^{\circ}-\left(24^{\circ}+\right.$ AÊD $)$
$\mathrm{BAAD}=66^{\circ}-\mathrm{AE} D$
Since AD is the plane of the celestial equator, at an angle with the horizon BD , which, in whatever latitude, is raised above the horizon at an angle equal to the complement of the

[^2]move from day to day along a plane at an angle with the plane of rotation of the earth, so that from day to day it appears a little higher or lower in the sky according to the part of the year. The difference in the figure for this inclination between antiquity and today is caused by the earth's nutation, or wobbling very slowly on its axis. The sun's declination ( $\delta$ ) is the angular measure of the position of the sun along this apparent path with the plane of the earth's rotation, the celestial equator. The figure for the declination thus passes from a negative maximum number of degrees south of the equator at the winter solstice to a positive maximum of the same magnitude north of it at the summer solstice. The points on the earth's surface at which the sun is then overhead at noon are called the Tropics.


Fig. 2.

| AD <br> cm | Latitude (BÂD) | AB <br> cm | BD <br> cm |
| :--- | :--- | :--- | :--- |
| 7.30 | $3 \mathrm{I}^{\circ} 56^{\prime}$ | 6.20 | 3.86 |
| 7.40 | $31^{\circ} 24^{\prime}$ | 6.32 | 3.86 |
| 7.50 | $30^{\circ} 52^{\prime}$ | 6.44 | 3.85 |
| 7.60 | $30^{\circ} \mathrm{I9}$ | 6.56 | 3.84 |
| 7.70 | $29^{\circ} 47^{\prime}$ | 6.68 | 3.82 |
| 7.80 | $29^{\circ} 14^{\prime}$ | 6.80 | 3.8 I |

## Table 2

latitude, and since $B \hat{A} D$ is the complement of $B \hat{D A}$, the latitude of the dial is therefore the same as BÂD.
II. To find the vertical height of the gnomon, $A B$, and the distance of $B$ from $D$ : In triangle ABD ,
$\mathrm{AB}=\mathrm{AD} \times \cos \mathrm{BÂ} D$
$\mathrm{BD}=\mathrm{AD} \times \sin \mathrm{BA} \mathrm{D}$
We may now apply these formulae to the six suggested values of $A D$ with the results given in table 2.

Given the uncertainties in the measurements from the stone, we may discard extreme results and say that the intended latitude for which the dial was constructed was between $29^{\circ} 47^{\prime}$ and $3 I^{\circ}$ $24^{\prime}$ and that the tip of the gnomon (A) was between 6.68 and 6.32 cm above a point $(\mathrm{B})$ on the dial which itself was between 3.82 and 3.86 cm south of point D . We may now reasonably take for purposes of illustration an average of these figures and say that the dial was constructed for latitude $31^{\circ}$ or just below and had a gnomon the effective height of which (even if the actual rod sloped and was longer) was about 6.5 cm and stood south of point D approximately 3.8 cm .

The true latitude of Aphrodisias is, however, $37^{\circ} 40^{\prime} \mathrm{N}$, something over six degrees of latitude further north than the latitude for which the dial was calibrated. Though the construction and marking out of the Aphrodisias dial was rather crude, as we have seen, there is little doubt that $3 \mathrm{I}^{\circ}$ was what the diallist intended. This can be demonstrated in FIG. 3 where the lines of the Aphrodisias dial (shown in black) are superimposed on two grids of lines for

horizontal dials showing the seasonal hours, one in latitude $3 \mathrm{I}^{\circ}$ (shown by dotted lines) and one for the approximate latitude of Aphrodisias, $38^{\circ}$ (shown by pecked lines).

The latter two grids have been constructed using modern trigonometric methods to find the azimuth of the sun to give the direction of the shadow of a gnomon at B , and the length of the shadow of a gnomon vertically above B, AB, of length $6.5 \mathrm{~cm} .{ }^{11}$ Fig. 3 is one fifth actual size. Since the ancients could not have used the precise trigonometrical methods available to us, and would have been forced to use geometry with less accurate means of measuring angles than we have ( $c f$. below), the figures chosen for the latitudes in the calculation have been rounded off to the nearest suitable degree which would have been used by the ancient diallists and the figure for the Inclination of the Ecliptic ( $\epsilon$ ) (hence for the maxima and minima of the sun's declination $\delta$ ) is again taken as $24^{\circ}$. These grids, like the Aphrodisias dial itself, have the curves for the sun's shadow-path at the summer solstice and the winter solstice and the straight line which it traces out at the equinoxes, along with the lines for the temporary or seasonal hours from I to iI-all the hours that can be shown on a dial of this type since at the hours o and I 2 , corresponding to sunrise and sunset, the sun is parallel with the plane of the dial and the shadow of the gnomon will run to infinity.

The most prominent difference between the grid for $31^{\circ}$ and that for $38^{\circ}$ is that the curve for the winter solstice is, at its least, on the noon (6) line, 2.94 cm further to the north of the foot of the gnomon, B , in latitude $38^{\circ}$ than it is in latitude $3 \mathrm{I}^{\circ}$. As we noticed above, the further north from the equator a place is, the lower will the sun be in the sky at a given moment. As a result the shadow of a vertical gnomon on a horizontal surface is longer at any particular time in a more northerly latitude than it is in a more southerly one. The difference is more noticeable with long shadows than with short ones. In trigonometrical terms this is because the cotangent of shallower angles (i.e. when the sun is low: the distance from $B$ to the end of the shadow of the gnomon $A B$ is found by the formula $6.5 \times$ cot. sun's altitude) increases faster with a small change of angle than it does with a similar small change amongst higher degrees. Thus at noon on the winter solstice

$\sin a=\sin \delta \cdot \sin \phi+\cos \delta \cdot \cos \phi \cdot \cos H$
$\cos A=\frac{\sin \delta-\sin \phi \cdot \sin a}{\cos \phi \cdot \cos a}$
where $a$ is the sun's altitude, $A$ its azimuth, $\delta$ its declination, $\phi$ the required latitude, and $H$ the hour angle (above, p. io3). To find the hour angle on any day, when the temporary or seasonal hour system is to be used, first find the hour angle of sunrise/sunset using the formula

$$
\cos \mathrm{h}=\tan \delta \cdot \tan \phi
$$

In all these formulae, when $\delta$ is to the south of the equator (from the autumn equinox to the winter solstice
and back to the spring equinox) its sine and tangent will be negative, with resulting negative effects on other parts of the formulae. Thus the hour angle ( $h$ ) of sunrise/sunset has a positive cosine in the winter, and so falls between $0^{\circ}$ and $90^{\circ}$, but a negative one in summer, putting it between $90^{\circ}$ and $180^{\circ}$. When the hour angle of sunrise/sunset has been found, this should be divided into six (the six hours from noon to the point in question). Each sixth part, or seasonal hour for the day in question, thus has an hour angle for use in the formulae above. Since hour angles are reckoned from the meridian (noon, hour 6 ) the sixth part itself will correspond with seasonal hours 5 and 7 , two-sixths with 4 and 6 , three sixths with 3 and 9 , etc. The azimuth quantity gives the direction of the shadow from the foot of the gnomon (here B); the vertical height of the gnomon (AB) multiplied by the cotangent of the altitude of the sun gives its distance from the foot of the gnomon. For the reader who is not a professional astronomer but is equipped with another aid not available to his ancient counterpart, the electronic calculator, a lucid account of how to calculate the position of the sun (though using the current value for the Inclination of the Ecliptic and the modern equatorial hour system) will be found in P. Duffet-Smith, Practical Astronomy with your Calculator (Cambridge 1979) 16-21, and esp. 24. A more sophisticated explanation will be found in the Explanatory Supplement to the Astronomical Ephemeris, etc. (London 1961, repr. 1974) ${ }^{24-6}$. On the change in the Inclination of the Ecliptic since antiquity (above, n. io) see ibid. 28.
the sun's altitude (still using the figure of $24^{\circ}$ for the sun's maximum southerly declination) is $35^{\circ}$ in latitude $3 \mathrm{I}^{\circ} \mathrm{N}$ but only $28^{\circ}$ in latitude $38^{\circ}$. The cotangents of these altitudes are $\mathrm{I} \cdot 428 \mathrm{I}$ and $\mathrm{I} \cdot 8807$. At the summer solstice the altitudes are $83^{\circ}$ and $76^{\circ}$ respectively. These give cotangents of 0.12279 and 0.24933 , which are numerically much closer to each other and result in much less difference in the shadow length of AB , as can be seen in Fig. 3 at the points where the winter solstice curve of the grids for the two latitudes crosses the meridian line (6).

Less pronounced, but still significant, is the difference in the hour lines. As we have seen, the further north one goes at any given time, the longer is the shadow of a gnomon of the same length on a horizontal sundial. Consequently the pecked hour lines for latitude $38^{\circ}$ are further from the noon line (6) than are the dotted ones for latitude $31^{\circ}$. For the same reason as the summer solstitial curves are further apart than are the winter ones for the two latitudes, the further the hour line is from the foot of the gnomon $B$ the greater the difference between the positions of the lines for the same hour in each locality.

It is very clear from fig. 3 that the lines the positions of which can be reproduced with reasonable certainty from the damaged Aphrodisias dial ${ }^{12}$ agree more closely with the grid for latitude $3 I^{\circ}$ marked in dotted lines than they do with the pecked lines for latitude $38^{\circ}$. The noon line will at all events agree, since the shadow of the sun at noon is cast due north on any day in any latitude. The equinoctial line in the Aphrodisias dial is less than a millimetre ( 0.07 cm ) short of the calculated equinoctial line for a dial in latitude $31^{\circ}$, and the hour line for the tenth hour on the stone coincides with the line for this hour calculated for latitude $31^{\circ}$. As for the other date lines, the winter solstitial on the stone is a fair fit with that required for latitude $31^{\circ}$, though it is distinctly different between the hours I and 2 and io and II. Because of the distance which the points for $I$ and $I I$ are from $B$ at the winter solstice, it is very easy for a dial maker to go wrong in drawing these without precision instruments, such as the ancient diallist would not have had. The shadow when it reaches this far from the gnomon is in any case fuzzy, because the sun, as a disc rather than a point of light, does not cast a sharp image, and this effect is exaggerated by the distance which the shadow has had to cover. The lack of definition of the end of the shadow means that the precise point at which the mark for I and II at the winter solstice should be put could not have been checked from mere observation. What remains of the summer solstitial curve on the stone suggests that this line was up to about 2.00 cm to the south of the calculated position for latitude $3 \mathrm{I}^{\circ}$, depending on the point of the line taken.

When we come to the hour lines, we find that $3,4,7,8$ and 9 are quite close to each other on the dial and grid for $31^{\circ}$ latitude. Hour 2 does not, however, match as does its theoretically symmetrical counterpart io. Both I and II are significantly out and do not correspond in the direction which they take. Since the lines for the dial in latitude $3 \mathrm{I}^{\circ}$ are set out in FIG. 3 as they should be, symmetrically about the meridian or noon (6) line, this Figure demonstrates graphically that in this respect at any rate the Aphrodisias dial was somewhat inexactly calibrated. However, in view of the agreement between the lines for the winter solstice and equinoxes in the two layouts, it cannot be that they are misaligned in Fig. 3. If the lines for the

[^3]|  |  | from BDE | from FDO |
| :--- | :---: | ---: | ---: |
| (cm) | Hour | (cm) <br> $(\mathrm{cm})$ |  |
| Winter solstice | 2 | $15 \cdot 10$ | 10.90 |
|  | 3 | 8.80 | 7.90 |
|  | 7 | 2.50 | 5.60 |
|  | 8 | $5 \cdot 00$ | 6.30 |
|  | 9 | 8.90 | 7.90 |
|  | 10 | 15.20 | 10.70 |
| Summer solstice | 1 | 24.00 | 14.80 |
|  | 10 | 11.50 | 8.00 |

The equatorial co-ordinates at the equinoxes are given above in table I . The broken lines in the grid from the dial in fig. 3 are reproduced from measurements from the squeeze.

Aphrodisias dial were shifted northwards so as to line up the summer solstitials, the hour lines would only move northwards, and not east and west, and would in consequence agree no better than they do now. On the other hand the other two date lines which do agree now would then cease to do so. It looks as though the hour lines are careless and the summer solstitial line on the dial has been cut too far to the south.

On the other hand the correct lines for this type of dial in latitude $38^{\circ}$, the latitude for which a diallist making a dial for Aphrodisias would probably have used, are quite different. The principal effect of the shift northwards in latitude is that the winter shadows, being cast by a sun which even in latitude $31^{\circ}$ did not rise higher than $35^{\circ}$ on midwinter's day and in latitude $38^{\circ}$ did not achieve more than $28^{\circ},{ }^{13}$ are considerably further from the base of the gnomon than either those of latitude $3 I^{\circ}$ or those of the Aphrodisias dial. The hour lines for latitude $38^{\circ}$ are likewise, being designed for longer shadows, further from the noon (6) line than those for the other two grids.

The effect of using a grid of lines apparently made for latitude $3 I^{\circ}$ or thereabouts in latitude $38^{\circ}$ would vary according to the time of year. In the present specimen at the summer solstice the shadow would fail to reach the curve so designated, which is appropriate either for a grid in a latitude nearer the equator, or for a shorter gnomon than the one which the rest of the lines show was used, or for a declination which the sun can never reach (almost $30^{\circ}$ ). In the midwinter the error would be more noticeable since the shadow would run along the date line for the winter solstice (at least from hour 2 to until hour 10 ), when the sun's declination was 16 or 17 degrees south, one day early in November. The shadow would then cross the curve labelled with the winter solstice and become longer each day until it reached the pecked line for latitude $38^{\circ}$. It would then begin to shorten until it again ran along the curve marked on the dial in the first three or four days of February, when the declination was again 16 or 17 degrees south. A very similar phenomenon would occur at the equinoxes. The shadow would be to the north of the line marked for the equinox at the spring equinox and would run along it about eighteen or nineteen days after the equinox. It would again reach this line about the same number of days before the autumn equinox, and be beyond it when the equinox came.

Anyone reading this dial could see that the date line was incorrect when the shadow fell outside the winter solstitial, but he would not realize so readily at other times of the year that the date was wrong, or, except from comparison with another sundial close at hand, that the hour shown was such that the instrument was slow in the morning and fast in the afternoon. If he had an appointment to keep and relied on this sundial, he would have been least unpunctual in the early afternoon or on mornings in the early spring or late autumn, when the discrepancies between the marked and correct lines were least. In practical terms, however, it is important to remember, in the diallist's favour, that the timekeeping would not have been much out. The whole scale of the dial is not large and errors in the hour lines are not easily perceptible without another, more accurate timepiece, such as a modern watch. The seasonal hour at the winter solstice in latitude $31^{\circ}$ is only 3.3 minutes longer than its counterpart seven degrees further north, and the summer solstitial hour is only a corresponding 3.3 minutes shorter. ${ }^{14} \mathrm{~A}$ dial of this size (as opposed to one twice its size), calibrated with lines of a fair width, will not be sensitive to a little over three minutes, which on the hours from 3 to 9 is not much different in linear distance on the grid from the thickness of the hour line itself. For everyday timekeeping the diallist had provided adequately.

The Aphrodisias dial thus has the grid of lines appropriate to a dial set up in latitude $3 \mathrm{I}^{\circ}$ which is in fact in northern Egypt. There are at least four ways of accounting for this. Firstly when we consider that the hour lines are not, as they should be, perfectly symmetrical, but that the morning hours to the west are somewhat displaced towards the noon line we might suspect

[^4]mins (winter), I h I 3.6 mins (summer). Gibbs (i7) believes that the marking of the date was less important in the Roman period than the record of the hours.

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an empirical construction on a block of stone which was set with a slight tilt towards the east. When we add the asymmetry of the winter solstice hours $I_{\text {and }}$ II , perhaps a little slope to the north might also be suspected. The effect of a tilt to the north-east would be to bring the morning hours closer to the noon line than they ought to be, and to exaggerate the length of midwinter shadows, especially at the end of the day. However, this does not account for the position of the summer solstice line, nor for the incorrect curve of the winter solstitial line, which could both have been marked better than they are from observation. The drawback with empirical calibration is, that while it almost ensures accuracy, it demands observation on three specific days (winter solstice, equinox and summer solstice) over a period of six months, and unimpaired sunlight all day on those days. The marks on the dial for the hours would have to be noted at times indicated by a separate dial. We may probably discount this method as too cumbersome.

A second solution is that the dial was made for a place, in latitude $31^{\circ}$ (on which more below) and then moved bodily to Aphrodisias for some cause. Such an incident is mentioned by Pliny, ${ }^{15}$ who says that a sundial captured as war booty in 264 B.c. from Catania in Sicily (latitude $37^{\circ} 3$ r $^{\prime}$ ) was used for many years as the official timepiece in the Forum at Rome (latitude $41^{\circ} 53^{\prime}$ ). In that case the discrepancy of latitude was less than the seven degrees in the case under scrutiny. Pliny says that though the hour lines were not correct the dial was used for ninety-nine years until a more accurate one was put beside it. Actually, though the error would have been observable, it would not have been as great as that expected in the Aphrodisias example. In the case of the Roman Forum we are probably dealing with a fairly small hemicyclium or hemispherium, ${ }^{16}$ but the Aphrodisias dial, though not very large, is cut into a hefty piece of local stone. Further we have no records or suspicion of plundering raids in late antiquity by the citizens of Aphrodisias and it is hard to see why they should have wanted to uproot an object so massive from Egypt when they could have had a more accurate dial cut back home.

The third possibility is that the former pagan altar which was to receive the dial was found by the diallist to be tilting about seven degrees to the south. This is a small, but quite perceptible slope, and rather than have the pillar altered, he calibrated the dial to compensate for the tilt. If a horizontal sundial for, say, latitude $40^{\circ}$ is taken to latitude $50^{\circ}$, it may be tilted ten degrees to the south and will then work perfectly well in its new latitude as a so-called direct south dial reclining $80^{\circ}$ from the vertical (or elevated $10^{\circ}$ from the horizontal). Many dials in the renaissance and later were made on this principle as travelling timepieces, using a plumbline to make the adjustment. Of course, if the dial was tilted to the south the winter shadows would not have been as long because the face of the dial was meeting them at a less shallow angle. The discrepancy in the winter solstitial and the hour lines could have been rectified in this way, and the equinoctial, which does not work for latitude $38^{\circ}$, would then coincide with the line for $31^{\circ}$ as shown in Fig. 3. However, the summer solstice line would have been wrong by only a little less than it was for latitude $38^{\circ}$ (cf. FIG. 3). This hypothesis is, however, the simplest and perhaps most likely.

The fourth explanation of the dial's construction is based on the difference in latitude. Of places with a latitude in or near $31^{\circ}$, one suggests itself as having plausible connections with dials and trade in the ancient world. This is Alexandria in Egypt (latitude $3 \mathrm{I}^{\circ} \mathrm{I} 3^{\prime}$ ). It is quite plausible that this city, where for instance Ptolemy, an astronomer par excellence, had lived and worked, could have been a home for a dial industry. It is possible that export dials were calibrated at Alexandria, though some expense would have been involved in handling the bulky finished objects. They could quite well have been cut elsewhere, as here at Aphrodisias, in situ from templates exported from Alexandria. Perhaps, even though it did not work as well as could be hoped in a different latitude, a dial calibrated for Alexandria had an illogical authority. If the

[^5]theory is right, it is curious that a template from Athens (latitude $38^{\circ}$ oo') or Rhodes (latitude $36^{\circ}$ $26^{\prime}$ ) which would have, especially in the former case, given a more accurate dial since the latitudes are almost the same, was not used. Perhaps these places did not deal in dials. Aquileia, Pompeii and Delos may have been exporting centres for finished sundials, ${ }^{17}$ though the evidence suggests that perhaps smaller types of completed dial than the present piece from Aphrodisias were generally involved. In our case the stone is that of the Aphrodisias region. It is asking too much to expect it to have been taken to Alexandria and shipped back after calibration. If the dial was of a mass-produced range rather than a 'one-off job', we might see in this another reason for less than perfect calibration. There is a great difference between accurate mathematical calculation of what should go on a dial and production of multiple copies.

Here, however, we might again consider the summer solstitial curve on the Aphrodisias dial. Did the diallist deliberately displace it? If so, did he do so with the knowledge that he was using a layout which was basically for Alexandria, but which he knew would have to be modified for his own latitude? If he knew his profession, he would have known that the biggest difference between the grid for Alexandria and that which he should have given for Aphrodisias was in the area around the winter solstice. To meet the requirements of Aphrodisias, or at any rate to give a roughly correct format of a dial calibrated for Alexandria, he would have had to have prolonged the hour lines in a northerly direction from the winter curve for Alexandria. Did he in fact mean to do this but erroneously make the extensions on the south side, working from the summer curve instead? There is no way of being sure, but the theory is attractive.

It is, of course, possible that the diallist used a combination of two or more of these methods. He may, for example, have put a gnomon into the stone, then observed a few points indicated by its shadow and then drawn in the geometric figure used by the ancients to delineate this type of dial and described by Vitruvius and Ptolemy under the name of analemma. ${ }^{18}$ This method is suggested by at least two other surviving examples of the pelecinum. ${ }^{19}$

Horizontal dials do not seem to have been prolific in antiquity as their counterparts have been in so many post-renaissance gardens in western Europe. This was probably for constructional reasons. Although the shadow of a stick in the ground appears to be the simplest form of timekeeper, the horizontal dial is more complex to mark off into the hour spaces for the temporary hour system than are the dials of spherical or conical section, which appear to have been more popular, ${ }^{20}$ since a basic understanding of the origins of the hyperbolic shadow paths on the plane surface is necessary in order to adapt the geometrical figure needed to make it. The diallist could very well have taken his pattern for a pelecinum from a dialling manual, but the greater effort required to cut a concave surface in the stone for the other types of dial is amply repaid by the comparative ease of marking that surface with equal spacings for the temporary or seasonal hours. The common hemicycle or hemisphere thus achieves an obvious advantage over the plane dial. The horizontal dial may have been considered worth making only for a public place where something larger is desirable: the concave dials can only comfortably be read from quite close up, and are generally on a smaller scale. ${ }^{21}$ For easy reading from a distance, the wall-dial, which the ancients also constructed, the principle of which is very little different from that of the horizontal dial, obviously has the lead over its horizontal brother in the agora. Some surviving examples of horizontal dials have the names of the winds surrounding the dial proper $^{22}$ and may have been used with or without a wind vane to indicate wind direction. One very much larger and very public timepiece (though that has eight vertical sundials) which

[^6]stands for use as a public dial on a shop roof on the Ponte Vecchio in Florence. See e.g. F. Cousins, Sundials (London 1969) 35.
${ }^{22}$ Gibbs, 87. On the wind-vane of the ancients, see A. Rehm, 'Antike Windrosen', Sitz. kön. Bay. Akad. Wiss. philos.-phil. u. hist. Kl. (1916) esp. si, 67.
unquestionably incorporated this purpose was the Tower of the Winds of Andronicus Cyrrhestes at Athens. ${ }^{23}$

The geometrical construction of this dial in antiquity was by the use of the analemma, which to the ancients meant a projection of the celestial sphere into one plane, from which in turn the positions of the hours on the dial's surface were deduced. Vitruvius ${ }^{24}$ describes the basic figure-omnium autem figurarum . . effectus unus-though his text at this point is somewhat obscure and he may well have not clearly understood what he was describing in any case; the way in which he dismisses the subject (sed ne multa scribendo offendam) tends to support this view. Ptolemy, in a work which presents even more textual difficulty, the $\pi \epsilon \rho i$ aj $\nu \lambda \bar{\epsilon} \epsilon \mu \mu \tau o s$, of which only palimpsest fragments of the original Greek survive ${ }^{25}$ (the rest of the text being supplied from the Latin translation printed by Commandinus ${ }^{26}$ ) elaborates the basic analemma by a variation of his own invention and envisages a dialling scale made in a durable material. ${ }^{27}$ Was the gnomonist of Aphrodisias using some such scale? Vitruvius states that any dial can be made from the analemma, though the details of the actual construction have been left to later commentators. The whole argument is set out most recently by Gibbs. ${ }^{28}$

The Aphrodisias dial does not seem to have been the most precise. We might recall that when describing his own (horribly crude) method of calibrating a sundial, Cetius Faventinus makes a remark which may have been in the mind of our Aphrodisian diallist:
est et alia de modo et mensuris horarum comparatio, quam prolixitatis causa praetereundam aestimavi, quoniam haec diligentia ad paucos prudentes pertinet. nam omnes fere . . . quota sit solum requirunt. ${ }^{29}$
The younger Seneca evidently suffered from the effects of this attitude: 'Horam non possum certam tibi dicere. facilius inter philosophos quam inter horologia conveniet. '30

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[^7]${ }^{26}$ Claudii Ptolemaei, Liber de Analemmate a Federico Commandino Urbinate instauratus et commentariis illustratus (Rome is62).

27 De Analemmate 1 I (p. 216 Heiberg); see also Gibbs 116.
${ }^{28}$ Gibbs, Appendix, ios-17.
${ }^{29}$ De Diversis Fabr. Arch. 29.
${ }^{30}$ Apocolocyntosis 2.
I should like to express my gratitude to Adrian Gratwick for his very searching comments on this article.
(a) Two Chian jars (new-style on left) from an Agora well, third quarter fifth century b.c. (Courtesy, American School of Classical Studies, Athens.) See Hesp. iv (1935) 496, fig. 17.
(b) Amphora stamp and silver didrachm of Chios, third quarter fifth century b.c. From Virginia Grace, Amphoras and the Ancient Wine Trade (1979), nos 48 f. (Courtesy, American School of Classical Studies, Athens.)
(c) Samian jar from sea near Samos. See V. Grace, Hesp. xl (1971) 73 f. and pl. i5, 4 (Courtesy, American School of Classical Studies, Athens). Scale i: io.
(d) Samian jar from a tomb at Marion (Courtesy, Swedish Cyprus Expedition, Stockholm). See E. Gjerstad, Swedish Cyprus Expedition ii (1935) 393 and pl. lxxv, I (centre pot). Scale i: io.
(e) Reverse of Samian trihemiobol, Barron no. 4 b (Oxford). See ( $j$ ) for acknowledgement. Scale $3 \times \mathrm{I}$.
(f) Samian jar from Lower Don cemetery. See J. Braschinsky, Krat. Sov. Inst. Arch. cxvi (1969) 114-17 with fig. 44,6. (Courtesy, Dr Braschinsky, who kindly sent the photograph.) Scale i: io.
(g) Samian jar from the Kerameikos. See U. Knigge, Kerameikos ix (1976) is i and pl. 64,8, Grave 288 (Courtesy, Deutsches Archaeologisches Institut, Athens.) Seale I: io.
(h) Reverse of Samian trihemiobol, Barron no. 3 a ( $B M_{120}$ ). For acknowledgement see ( $j$ ). Scale $3 \times \mathrm{I}$.
(i) Samian jar from Olbia. See Braschinsky, Krat. Sov. Inst. Arch. cix (1967) 22-4 with fig. 2 and Archaeologia xix (Warsaw 1968), 55-7 with fig. I2. (Courtesy, Dr Braschinsky, who again provided the good photograph.) Scale I: 10 .
( $j$ ) Reverse of Samian trihemiobol, Barron 2 a (Berlin). Scale $3 \times$ I. The coins $(e),(h)$ and $(j)$ are reproduced from Hesp. xl (1971) pl. 15,6-8. (Courtesy, American School of Classical Studies, Athens.)
(k) Samian jar from Thasos. It was briefly published in $B C H \operatorname{lxxv}^{(1951)} \mathrm{I} 79 \mathrm{f}$. with fig. 98. (Courtesy, École Française d'Athènes, to whom I owe my photograph.) Scale i : io.
(l) Reverse of Samian tetradrachm, Barron Class III no. 39 b (Cambridge). Scale $3 \times$ I. (Courtesy, Prof. Barron, from whose pl. x my photograph comes.)


The Aphrodisias Sundial, from the South. (Courtesy, Prof. K. T. Erim)


[^0]:    ${ }^{1}$ This information together with the assessment of the date and the photograph comes from the excavator Professor K. T. Erim, to whom I am most grateful. Thanks are especially due to Miss J. M. Reynolds for making a squeeze and for drawing my attention to the dial.
    ${ }^{2}$ Hyperbola: the curve formed when a plane cuts a cone at an angle greater than the slope of the cone. In the case of a horizontal sundial the point of the cone is the tip of the gnomon and the cone itself is generated by the sun's apparent movement during the day around this point. The circular base of the cone is thus in a plane parallel with the plane in which the sun seems to be

[^1]:    ${ }^{9}$ These measurements were taken from a squeeze soon after it was made, and have been checked on site by

    Mossman Roueché. There may still be errors, since I
    have not seen the actual stone.

[^2]:    ${ }^{10}$ Vitruv. De Arch. ix 7.2.4 ('one fifteenth of a circle'): see T. L. Heath, Aristarchus of Samos (Oxford 1913) I3I n. 4 for ancient opinions on the question. The true value was $23^{\circ} 4 I^{\prime} 7^{\prime \prime}$ in the time of Ptolemy (second century A.D.) $R E$ s.v. Ekliptik, v (1905) 2208-1 3 , esp. 2212 (A. Rehm). Today it is $23^{\circ} 26^{\prime} 51^{\prime \prime}$. On the subject see O. Neugebauer, A History of Ancient Mathematical Astronomy (Berlin 1899; New York 1975) ii 733-4, esp. 734 n. II. The Inclination of the Ecliptic or great circle of the heavens in which the sun appears to move (and hence in which eclipses always occur when the moon crosses this plane) is actually the angle at which the earth's axis of rotation is inclined to the plane of the orbit in which the whole planet moves around the sun. Looked at geocentrically this makes the sun appear to

[^3]:    ${ }^{12}$ The hour points on the three seasonal curves can be fixed from co-ordinates taken from the squeeze and checked on the stone by Mossman Roueché. The equinoctial line on the dial (FDO in FIG. I) may be taken as the $x$ axis and the meridian or 6 line ( BDE ) as the $y$ axis. The hour points on the stone may then be expressed as distances from perpendicular to BDE ( $x$ co-ordinates) and perpendicular to FDO ( $y$ coordinates) as follows

[^4]:    ${ }^{13}$ Still taking $\epsilon=24^{\circ}\left(\delta=-24^{\circ}\right)$.
    14 In equatorial hours, etc., these times are: lat. $3 \mathrm{I}^{\circ}$ 49.7 mins (winter), i h io. 3 mins (summer); lat. $38^{\circ} 46 \cdot 4$

[^5]:    ${ }^{15}$ Pliny NH vii 60 (214) ff., cf. Censorinus De Die $\quad{ }^{16}$ See Gibbs 12-39, 122-322. Natali 23.

[^6]:    ${ }^{17}$ Cf. Gibbs 71, 90-1.
    18 Vitruvius ix 7; Ptolemy, $\pi \epsilon \rho i{ }^{\text {à }} \boldsymbol{\nu} a \lambda \not{ }_{\eta} \mu \mu a \tau o s$ (ed. Heiberg 1907).
    ${ }^{19} C f$. Gibbs $79-80$ and cat. nos. $4004 \mathrm{G}, 4005 \mathrm{G}$.
    ${ }^{20}$ Gibbs, 66, 73, 78.
    ${ }^{21}$ There are exceptions, e.g. Gibbs, cat. no. 3008G, p. 227 and pl. 28, p. 228; and a medieval hemicycle still

[^7]:    ${ }^{23}$ See J. Stuart and N. Revett, The Antiquities of Athens (London 1762) i ch. 3, pls I, III, =J. Travlos, Pictorial Dictionary of Ancient Athens (London 1971) 283; Gibbs 342-5.
    ${ }^{24}$ De Arch. ix 77.
    ${ }^{25}$ Claudii Ptolemaei, Opera, ed. J. L. Heiberg (Leipzig 1907) ii. The palimpsest fragments of the Greek are from manuscript Ámbrosianus Gr. L 99 (saec. VII); the thirteenth-century Latin translation by William of Moerbeke (from the autograph archetype, Vat. Ottob. Lat. I850). The Greek text may have disappeared at the time of the Fourth Crusade.

